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On the Rate of Interest in Loans repayable by Instalments. By PETER GRAY, F.R.A.S., *Honorary Member of the Institute of Actuaries.*

THERE must at the present time be a vast amount of British capital embarked in loans. Foreign states are constantly holding out their lures with more or less of success to our monied men, and financial and other associations are always ready to take charge of the funds of such of them as prefer investing at home. In this state of matters it is somewhat remarkable that there is nothing to be found in our books on interest on the subject of loans. These are usually—and perhaps intentionally—so complicated with conditions in regard to premiums, discounts, times and modes of repayment, &c., as to render it almost always a matter of extreme nicety to determine the rate paid by the borrower for the accommodation, and that realized by the lenders on their investments.* And yet, as I have just said, in no English work that I am aware of, is there anything to be found having special reference to the subject.

It is of course desirable, in the interests of both borrowers and lenders, that the rates involved in the transactions into which they enter should be pretty accurately known; and doubtless those parties form conclusions for themselves upon the points in question. There is however reason to fear (and ground for this opinion will hereafter be shown) that the conclusions thus arrived at are not always trustworthy, nor such as will be borne out by the results. Now I find that in most, if not all, cases, it is quite practicable to determine the exact rates, at the cost of no great expenditure of time or trouble; and I have thought it might be useful to devote a paper to the subject.

The method I propose is simply to treat the repayments as an annuity whose present value is the sum advanced; and the problem is thus reduced to finding the rate of interest involved in the annuity. It is true that the annuities with which we shall thus have to deal will be, mostly, very different from uniform annuities. Still, in point of fact, it will usually, if not always, be found, that the payments are regulated by laws which may be discovered, and the discovery of which renders them amenable to the resources of analysis. In the present paper I shall illustrate the method of treatment I propose by applying it to two examples.

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* These two rates will be the same in regard to a specified loan if the whole of the loan is held by a single individual; but by no means necessarily so if it is held by more than one. This will be seen hereafter.

I have said that I am not aware that the subject of loans is treated in any English work. A French work however, of which I give the title below,* has recently come into my hands, in which it is taken up. It consists of 128 pages of introductory matter and 129 pages of tables. The principal tables are the amount of one franc (or one pound), the amount of an annuity of one franc, and the annuity that one franc will purchase, all for every number of years from one to one hundred, and at no fewer than eighty-five rates of interest, extending from one to ten per cent. No tables approaching these in the number of rates exhibited have hitherto been published. I mention this work here because my first example is the most complex of those solved by M. Violeine, and I shall have occasion to remark on the method he employs, which I consider to be founded on an altogether erroneous principle.

Example 1.†—A loan of £10,000,000 is contracted at 3 per cent.; and the debt is represented by 10,000 bonds, nominally of £1,000 each. These are to be paid off with a premium of 25 per cent., as follows:—94 the first year, 102 the second, 110 the third, and so on, increasing by 8 each year till the fortieth, when the number paid off will be 406, completing the number of 10,000. It is required to determine the rate per cent. that the loan costs the borrower, and also the rate that will be realized by the holders of the bonds.

Here it will be remarked that although the interest to be paid is 3 per cent., the cost of the loan to the borrower is much more. The charge for each year consists, in addition to the interest on the amount due at the beginning, of a premium of 25 per cent. on the portion of principal paid off at the end of it.

The conditions of the loan might have been stated differently. The loan of £10,000,000 is here issued at par, and the sum lent is to be returned with a premium of 25 per cent. But the loan might have been called one of £12,500,000 issued at 20 per cent. discount, bearing interest at the rate of 2·4 per cent., (125 : 120 :: 3 : 2·4) and repayable at par. But borrowers are wise in their generation. There is little doubt that although the two schemes are identical, to many lenders the first would prove more attractive than the second. There is a charm in the prospect of getting back one's capital not only intact, but increased by 25 per cent., (a moderate interest being paid upon it while withheld,) which would not be found in a discount of 20 per cent. at the outset, accompanied as it is by an abatement of 0·6 per cent. in the rate of interest allowed, and repayment at par.

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* *Nouvelles Tables pour les Calculs d'Intérêts Simples et Composés, d'Amortissement, d'Annuités de Primes, etc. Par P.-A. Violeine. A Vaugirard*, 1854. 4to. pp. 128 and 130. I may add that this work appears to be a recognised authority for the purposes of the *Credit Foncier*. A copy of it, I learn, has been recently added to the Library of the Institute of Actuaries.

† Violeine, p. 124. M. V. has *francs*. I use *pounds*, as a measure of value with which we are more familiar.

It may be here remarked also, that the object, and the effect, of the annual increase in the number of bonds paid off, is to equalize, in a measure, the annual payments of the borrower. As the debt is reduced, the interest payable decreases; and the power of redemption, out of a uniform or nearly uniform revenue, consequently increases. Although it is no part of the *quæsita* of the problem to show how, with the end which has just been specified in view, the particular gradation 94, 102, &c., has been obtained, it may nevertheless be of use to point this out.

To pay off £10,000,000 in forty years, at say 3 per cent., we find by reference to the tables, an annual payment of £432,624 would be necessary. Consequently, £300,000 being the interest payable the first year, £132,624 remains available for the reduction of the principal; and this would suffice to pay off about 106 bonds, at £1,250 each.[*] Let a be the number of bonds to be paid off the first year, and d the annual increase, Then the sum of forty terms of the series, a, $a+d$, $a+2d$, ... is, by a known theorem,

$$40a+\frac{40\times 39}{2}d=40a+780d.$$

Equating this to 10,000, the entire number of bonds, we get,

$$a=250-19\tfrac{1}{2}d.$$

We must here substitute for d the least *even* number that will give for a a value not exceeding 106. This is easily found to be 8; whence $a=94$, and the series is 94, 102, 110 . . . . 406.

To proceed now to the solution, first as regards the cost of the loan to the borrower. I have elsewhere shown[†] that if b_1, b_2, b_3, &c., denote the successive payments of an annuity, the present value of that annuity for n years, will be,

$$\frac{b_1}{i}+\frac{\Delta b_1}{i^2}+\frac{\Delta^2 b_1}{i^3}+\ldots-v^n\left(\frac{b_{n+1}}{i}+\frac{\Delta b_{n+1}}{i^2}+\frac{\Delta^2 b_{n+1}}{i^3}+\ldots\right),\ddagger\ \ldots\ (1.)$$

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[*] The above is a very rude approximate process.

[†] *Assurance Magazine*, vol. vi., p. 191. See also Introduction to the re-issue of Orchard's *Assurance Premiums*, pp. 12, 13.

[‡] The above expression is simply the finite integral of $b_n v^n$. The general form is,

$$\Sigma b_n v^n=\mathrm{C}+\frac{b_1}{i}+\frac{\Delta b_1}{i^2}+\frac{\Delta^2 b_1}{i^3}+\ldots$$

For $\mathrm{C}=b_0$, this gives the present value of the annuity to infinity when the first payment is made now; and for $\mathrm{C}=0$, it gives the like when the first payment is made a year where i is the rate of interest per pound, and $v=(1+i)^{-1}$, as usual. And this will obviously be a finite expression if b_m be a rational and integer function of m, since in that case the differences of b_1, and b_{n+1}, will ultimately vanish.

This theorem will serve our present purpose. We shall have first to determine the law of the annuity in order to be able to assign the values of the symbols, b_1, Δb_1, &c.; then, substituting these in (1), to equate the result to 10,000,000, the given present value. The value of i deduced from the equation so formed, will be the rate that the loan costs the borrower.

To determine the law of the annuity we must form the numerical values of the first few of its terms, as follows:—

		10,000,000	
Int. at 3 per cent.	300,000		
Pm. on 94 Bds.	23,500	323,500	Charge, 1st year
		10,323,500	
Int.	300,000		
94 Bds. . . .	117,500	417,500	Payment, 1st year, $=b_1$
		9,906,000	
Int.	297,180		
Pm. on 102 Bds.	25,500	322,680	Charge, 2nd year
		10,228,680	
Int.	297,180		
102 Bds. . .	127,500	424,680	Payment, 2nd year, $=b_2$
		9,804,000	
Int.	294,120		
Pm. on 110 Bds.	27,500	321,620	Charge, 3rd year
		10,125,620	
Int.	294,120		
110 Bds. . .	137,500	431,620	Payment, 3rd year, $=b_3$
		9,694,000	
Int.	290,820		
Pm. on 118 Bds.	29,500	320,320	Charge, 4th year
		10,014,320	
Int.	290,820		
118 Bds. . .	147,500	438,320	Payment, 4th year, $=b_4$
		9,576,000	

---

hence. The form in the text is that which arises when the integral is taken between the limits 1 and $n+1$. Multiplication of it by $(1+i)^n$ converts it into

$$(1+i)^n\left(\frac{b_1}{i}+\frac{\Delta b_1}{i^2}+\dots\right)-\left(\frac{b_{n+1}}{i}+\frac{\Delta b_{n+1}}{i^2}+\dots\right)$$

which is the *amount* of the annuity, first payment a year hence, in n years.

Differencing the above values of b_1, b_2, &c., as below:—

$$\begin{array}{l|l|l|l}
b_1=417500 & 7180 & -240 & 0 \\
b_2=424680 & 6940 & -240 & \\
b_3=431620 & 6700 & & \\
b_4=438320 & & &
\end{array}$$

we have, $b_1=417500$, $\Delta b_1=7180$, $\Delta^2 b_1=-240$, $\Delta^3 b_1=0$ . . . Now we know that

$$b_{1+m}=b_1+m\Delta b_1+\frac{m(m-1)}{2}\Delta^2 b_1+\ldots$$

therefore, putting in the above values,

$$b_{1+m}=417500+7300m-120m^2;$$

and finally, changing m into $m-1$,

$$b_m=410080+7540m-120m^2.$$

This is the general expression for the mth payment; but we shall find that the expression for the $(1+m)$th will answer our purpose rather more conveniently.

The theorem (1) for $n=40$ becomes,

$$\frac{b_1}{i}+\frac{\Delta b_1}{i^2}+\frac{\Delta^2 b_1}{i^3}-v^{40}\left(\frac{b_{41}}{i}+\frac{\Delta b_{41}}{i^2}+\frac{\Delta^2 b_{41}}{i^3}\right);$$

from which it appears that having formed b_1, Δb_1 and $\Delta^2 b_1$, we have still to form b_{41}, Δb_{41} and $\Delta^2 b_{41}$.

We substitute 40, 41 and 42 successively, for m, in b_{1+m}, as follows:—*

$$\begin{array}{rrl}
-120 & 7300 & 417500(40 \\
 & 2500 & 517500 \;=b_{41} \\
 & -2300 & \\
\hline
-120 & -2300 & 517500(1 \\
 & -2420 & 515080 \;=b_{42} \\
 & -2540 & \\
\hline
-120 & -2540 & 515080(1 \\
 & -2660 & 512420 \;=b_{43}
\end{array}$$

We have in this process b_{41} and Δb_{41}, and we already know that $\Delta^2 b_{41}=-240$. For distinctness however I determine them as follows:—

$$\begin{array}{l|l|l}
b_{41}=517500 & -2420 & -240 \\
b_{42}=515080 & -2660 & \\
b_{43}=512420 & &
\end{array}$$

We therefore have

$$b_{41}=517500,\ \Delta b_{41}=-2420, \text{ and } \Delta^2 b_{41}=-240.$$

Substituting now in (1) we have for determining the value of i, the equation,

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* See *Journal of the Institute*, vol. xiii., p. 64.

$$\frac{417500}{i}+\frac{7180}{i^2}-\frac{240}{i^3}-v^{40}\left(\frac{517500}{i}-\frac{2420}{i^2}-\frac{240}{i^3}\right)=10{,}000{,}000.$$

There being no direct method of solving this equation, the solution must be effected by trial; and in this we shall be aided by the use of logarithms. For this purpose I put the expression into the following more convenient although less elegant form:—

$$\frac{417500}{i}+\frac{7180}{i^2}-\frac{240}{i^3}$$
$$-\frac{517500}{i(1+i)^{40}}+\frac{2420}{i^2(1+i)^{40}}+\frac{240}{i^3(1+i)^{40}}=10{,}000{,}000.$$

The operation is as follows:—

	i	·05	·04	·0368	·03727	·0372585
(1)	log i	$\bar{2}$·6989700	$\bar{2}$·6020600	$\bar{2}$·5658478	$\bar{2}$·5713594	$\bar{2}$·5712254
(2)	„ i^2	$\bar{3}$·3979400	$\bar{3}$·2041200	$\bar{3}$·1316956	$\bar{3}$·1427188	$\bar{3}$·1424508
(3)	„ i^3	$\bar{4}$·0969100	$\bar{5}$·8061800	$\bar{5}$·6975434	$\bar{5}$·7140782	$\bar{5}$·7136762
	„ $(1+i)^{10}$	0·2118930	0·1703334	0·1569499	0·1589182	0·1588699
	„ $(1+i)^{40}$	0·8475720	0·6813336	0·6277996	0·6356728	0·6354796
(4)	„ $[i(1+i)^{40}]$	$\bar{1}$·5465420	$\bar{1}$·2833936	$\bar{1}$·1936474	$\bar{1}$·2070322	$\bar{1}$·2067050
(5)	„ $[i^2(1+i)^{40}]$	$\bar{2}$·2455120	$\bar{3}$·8854536	$\bar{3}$·7594952	$\bar{3}$·7783916	$\bar{3}$·7779304
(6)	„ $[i^3(1+i)^{40}]$	$\bar{4}$·9444820	$\bar{4}$·4875136	$\bar{4}$·3253430	$\bar{4}$·3497510	$\bar{4}$·3491558
	„ 417500	5·6206565	5·6206565	5·6206565	5·6206565	5·6206565
	(1)	$\bar{2}$·6989700	$\bar{2}$·6020600	$\bar{2}$·5658478	$\bar{2}$·5713594	$\bar{2}$·5712254
	A	6·9216865	7·0185965	7·0548087	7·0492971	7·0494311
	„ 7180	3·8561244	3·8561244	3·8561244	3·8561244	3·8561244
	(2)	$\bar{3}$·3979400	$\bar{3}$·2041200	$\bar{3}$·1316956	$\bar{3}$·1427188	$\bar{3}$·1424508
	B	6·4581844	6·6520044	6·7244288	6·7134056	6·7136736
	„ 240	2·3802112	2·3802112	2·3802112	2·3802112	2·3802112
	(3)	$\bar{4}$·0969100	$\bar{5}$·8061800	$\bar{5}$·6975434	$\bar{5}$·7140782	$\bar{5}$·7136762
	C	6·2833012	6·5740312	6·6826678	6·6661330	6·6665350
	„ 517500	5·7139104	5·7139104	5.7139104	5·7139104	5·7139104
	(4)	$\bar{1}$·5465420	$\bar{1}$·2833936	$\bar{1}$·1936474	$\bar{1}$·2070322	$\bar{1}$·2067050
	D	6·1673684	6·4305168	6·5202630	6·5068782	6·5072054
	„ 2420	3·3838154	3·3838154	3·3838154	3·3838154	3·3838154
	(5)	$\bar{2}$·2455120	$\bar{3}$·8854536	$\bar{3}$·7594952	$\bar{3}$·7783916	$\bar{3}$·7779304
	E	5·1383034	5·4983618	5·6243202	5·6054238	5·6058850
	„ 240	2·3802112	2·3802112	2·3802112	2·3802112	2·3802112
	(6)	$\bar{4}$·9444820	$\bar{4}$·4875136	$\bar{4}$·3253430	$\bar{4}$·3497510	$\bar{4}$·3491558
	F	5·4357292	5·8926976	6·0548682	6·0304602	6·0310554

A	8350000	10437500	11345109	11202041	11205496
B	2872000	4487500	5301867	5168989	5172180
C	1920000	3750000	4815793	4635888	4640182
D	1470200	2694700	3313317	3212759	3215181
E	137500	315000	421038	403110	403539
F	272700	781100	1134666	1072655	1074126
	11632200	16021100	18202680	17846795	17855341
	3390200	6444700	8129110	7848647	7855363
Result	8242000	9576400	10073570	9998148	9999978

The scheme exhibits the details of five trials, each occupying a column, with their results. Each column is headed with the value of *i* used in that column. The first portion of the column is occupied with the formation of the logarithms of the denominators of the several terms; the second portion shews the formation of the logarithms (marked A, B . . . . F) of the terms; and the third portion exhibits the combination of the numbers corresponding to those logarithms for the production of the result, which occupies the last line. The positive and the negative terms are distinguished from each other by being printed in different type.

I commence my trials, pretty much at random, with 5 per cent. This is found to be too great, the result, 8242 . . . , being too small. I try 4 per cent.; but this too appears to be too great, although nearer the truth than 5 per cent. I now apply the rule of False Position, by assuming that variation in the rate is approximately proportional to variation in the result.

Thus:—

95764 100000
82420 95764

13344 : 4236 :: —·01 : —·0032.

We thus find that since a variation of —·01 in the rate produces in the result a variation of 13344, therefore 4236, the required variation in the result, will be produced by a variation in the rate of —·0032. Hence ·04—·0032=·0368 is the next rate to be tried. This is found to be nearer the truth, but too small. A new correction is then obtained as follows:—

100735 100000
95764 100736

4972 : —736 :: —·0032 : ·00047.

Hence ·0368+·00047=·03727 is the corrected rate. This, as appears by the result, is a little too great. We therefore seek another correction, as follows:—

$$\begin{array}{rr} 999815 & 1000000 \\ 1007357 & 999815 \\ \hline -7542 \ : & 185 \ :: \ \cdot00047 \ : \ -\cdot0000115. \end{array}$$

And ·03727 − ·0000115 = ·0372585 is found on trial to be the rate far more than sufficiently near to the truth to serve any practical purpose.

We conclude then that 3·72585, or, rather more nearly, 3·72584, is the rate per cent. that the loan costs the borrower; or, in other words, that it is the rate at which the sum borrowed must be improved, so as just to provide for the stipulated payments as they fall due.

I may here add that the foregoing result has been fully verified by a different method—in fact it was obtained before the problem was brought under my notice—by my friend Colonel Oakes.

Instead of the value just found, M. Violeine assigns as the exact rate—*le taux exact*—3·7613, which differs from it materially. He effects his solution by means of a general formula, of which he offers no demonstration. Here it is:—

$$\text{“R}= \frac{(pr+af+aft)\dfrac{b^n-1}{b-1}-\dfrac{afr-df(1+t)}{b-1}\left(\dfrac{b^n-1}{b-1}-n\right)-df\left\{\dfrac{1}{b-1}\left[\dfrac{b^n-1}{b-1}-b-(n-1)\right]-\dfrac{n^2-n-2}{2}\right\}^{*}}{\dfrac{b^n-1}{b-1}}$$

“ *dans laquelle*
“ p =*la somme empruntée,* . . . . [10,000,000]
“ r =*le taux de l'intérêt °/ₒ*
“ $b=1+\dfrac{r}{100}$,
“ f =*la valeur d'une action ou obligation,* . . . . . [1,000]
“ a =*nombre d'actions remboursées la première année,* . . [94]
“ d =*la raison de la progression,* . . . . [8]
“ t =*taux °/ₒ de la prime,* . . . . [25]
“ R=*paiement annuel.*”

On this it is necessary to remark at the outset, that although r and t are here defined as rates *per cent.*, they are used in the formula as rates *per pound.* Also, n, the number of years, is omitted from the enumeration.

Now it will be found that on inserting the values belonging to the present problem (which I have placed opposite the several symbols), and any assumed value of *r*, the numerator of this singular expression gives the amount at the end of the term, 40 years, at the assumed rate, of the variable annuity, that is of the payments that have to be made by the borrower; and the denominator is the amount in the same time of an annuity of £1. Hence the entire expression, the value of R—the *paiement annuel*, as M. V. calls it—is the uniform annuity, at the same rate and during the same term, that is equivalent to the variable annuity. Now we know that the variable annuity amortizes—redeems, pays off—the loan in 40 years at a certain rate of interest; and consequently, by what has just been shewn, the value of R corresponding to that rate, would be the uniform annuity which would amortize the loan in the same time. Hence if we know this rate we shall be able, if we please, to determine R, the uniform amortizing annuity. But we do not know this rate. It is in fact the very rate we are in search of; and all we know about it is that it is greater than 3 per cent., since, in addition to interest at this rate, it has to include the equivalent of the premium of 25 per cent. on the sum borrowed. And therefore we cannot (in this way) arrive at the value of R. M. Violeine is not arrested by this difficulty. He assumes 3 per cent. for the rate, and calls the resulting value of R, 487,424, the uniform amortizing annuity. He then finds (by a method that I need not here stop to describe) that the annuity 487,424 will amortize 10,000,000, the sum borrowed, in 40 years at 3·7613 per cent;—in other words, that 10,000,000 is the value of an annuity of 487,424 for 40 years at 3·7613 per cent. And therefore he assigns this rate as the rate required: not an approximation to it, but the *exact* rate—*le taux exact.*

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* Surely the first term of this numerator were better written $\{pr+af(1+t)\}$, and the last $\dfrac{(n-2)(n+1)}{2}$

It is not easy to unravel the tangle here. It is however plain enough that, 487,424 not being the correct amortizing annuity, the rate found cannot be the true rate; and the valuing of the annuity at two different rates is entirely analogous to giving different values to the same symbol on opposite sides of an equation. One is in fact irresistibly reminded of Hogarth's well known illustration of false perspective, in which we see, amongst other incongruities, a man on a hill in the distance lighting his pipe at a candle held by a woman from the first (or second) floor window of a house in the foreground.

I now proceed to the second requirement of the problem, namely, the determination of the rate of interest that will be realized by the lenders. Were there but one lender—were the whole of the bonds held by a single individual—the rate he would realize would obviously be, as already remarked, that paid by the borrower. But the bonds are not, either necessarily, or in point of fact, so held. There *may* be as many lenders as there are bonds; and the rate realized on each bond will depend on the period at which it is paid off, since it is then that the premium becomes available. What we have to do therefore is to find the rate that will be realized on a bond according as it falls to be paid off in the first, second, third, &c., year. I have omitted to mention that the bonds to be paid off each year are selected by lot; and as the rate varies between wide limits—being no less than 28 per cent. on the bonds paid off in the first year, while it is but a little over 3 per cent. on those paid off in the last—it is quite conceivable that this element of uncertainty—embracing as it does the *possibility* of a very large return on the sum invested—may act with no small effect as a lure on minds of a certain constitution.

Take a bond which will be paid off in the nth year. In return for his £1000 invested now the holder receives an annuity of £30, for n years, and a sum of £1250 at the end of the nth year. The present value of the annuity, at i per pound, is

$$\frac{30(1-v^n)}{i},$$

and that of the sum is, $1250v^n$.

Hence $$\frac{30(1-v^n)}{i}+1250v^n=1000,$$

or $$\frac{3-3v^n+125iv^n}{i}=100,$$

or $$\frac{3+(125i-3)v^n}{100i}=1.$$

And from this equation we shall have to determine the values of i for every value of n from 1 to 40.

Like the former equation the present can only be solved by trial; but although the labour of doing so in any particular case is not great, I have not thought it worth while to encounter it in more than four instances, viz., for $n=10$, 20, 30, and 40. For $n=1$ we have $100i=28$ independently of the theorem. In the following table the first column contains the rates per cent. as determined by the theorem, and the second contains the rates assigned by M. Violeine for the same values of n.

1	28·	28·
10	4·989	4·77
20	3·8526	3·67
30	3·495	3·34
40	3·307	3·19

For illustration I subjoin the operation for $n=20$.

$100i$	4·00	3·5	3·857	3·8526
$125i$	5·00	4·375	4·82125	4·81575
$125i-3$	2·00	1·375	1·82125	1·81575
log $(1+i)^{10}$	0·1703334	0·1494035	0·1643577	0·1641738
„ $(1+i)^{20}$	0·3406668	0·2988070	0·3287154	0·3283476
„ v^{20}	$\bar{1}$·6593332	$\bar{1}$·7011930	$\bar{1}$·6712846	$\bar{1}$·6716524
„ $(125i-3)$	0·3010300	0·1383027	0·2603696	0·2590561
Sum	$\bar{1}$·9603632	$\bar{1}$·8394957	$\bar{1}$·9316542	$\bar{1}$·9307085
$(125i-3)v^{20}$	·912774	·691028	·854386	·852528
$3+(125i-3)v^{20}$	3·912774	3·691028	3·854386	1·852528
log „	0·5924849	0·5671474	0·5859552	0·5857458
„ $100i$	0·6020600	0·5440680	0·5862496	0·5857539
Diff.	$\bar{1}$·9904249	0·0230794	$\bar{1}$·9997056	$\bar{1}$·9999919
Result.	·97819	1·05458	·99932	·9999815

The principle of this process is the same as that already employed. We first try $i=·04$, which is found to be too great. We then try $i=·035$, which is too small. A correction to this last value is then found thus:—

1·05458	1·00000
·97819	1·05458
·07639	—·05458

$$·07639 : -·05458 :: -·005 : ·00357.$$

We thus have $·035+·00357=·03857$ for a new value, which is too great, and we seek another correction as follows:—

·99932	1·00000
1·05458	·99932
—·05526	·00068

$$-·05526 : ·00068 :: ·00357 : -·0000439.$$

This gives $i=·03857-·000044=·038526$, which is very near the truth, the error being $1-·9999815=·0000185$. I have, for exemplification of the principle, gone much further than is necessary. The value ·03857 is quite near enough, and that was reached by two operations.

It will have been perceived that in the solution of this part of the problem, as in that of the former, I am at variance with M. Violeine. His results, except in the case of $n=1$, are always less than mine. Let us attend then to his method of solution. It is so exceedingly simple that it will be matter for regret to find that it gives erroneous results. M. Violeine here, as in the former case, selects the end of the term, (that is, the date of the payment of a bond) as the epoch of comparison; being probably led to this by the circumstance that his tables have reference to *amounts,* and not *present values.* The bondholder having during the term received annual interest at 3 per cent., and being, at the end of that period, put in possession of the principal with the premium upon it, M. V. considers that the value of the whole at that date is, first, the amount of the bond improved at 3 per cent. for n years, viz., $1000(1 \cdot 03)^n$; and secondly, the premium just paid, viz., £250. So that, calling i the required rate, we have

$$1000(1+i)^n = 1000(1 \cdot 03)^n + 250,$$

or,

$$(1+i)^n = (1 \cdot 03)^n + \cdot 25;$$

whence,

$$\log(1+i) = \frac{\log\{(1 \cdot 03)^n + \cdot 25\}}{n},$$

And from this equation the value of i for every value of n can be very readily determined.

M. Violeine has determined the whole of the values; and I have selected five of those which he gives for comparison with the corresponding values deduced by my process. With the exception of the first, which is the same, M. V.'s values are always less than mine. And the reason is that in his process it is implied that the payments of the annuity as received are forthwith invested at 3 per cent., while in my process no such unwarranted assumption is necessary. Money moreover is, by the terms of the problem, worth more than 3 per cent. To procure the required accommodation *nominally* at 3 per cent., the borrower, in addition to this rate, has to pay also a premium of 25 per cent. on the amount lent.

On the whole, in the view of M. Violeine's method of going to work, I think I may venture to remark, that if our Gallican neighbours .have been the first to enter on this new field, their success in the cultivation of it has not hitherto been such as need unduly excite our national susceptibilities.

The second example I had in view was the Austrian Loan of 1865; but in consequence of the space already occupied the consideration of this must be deferred.

Conditions of Assurance.

FOR many years past there has been a continual tendency on the part of our Life Insurance Companies to make the conditions of their policies as to foreign residence and travel more simple and more liberal. Formerly, persons whose lives were insured were not allowed to proceed out of Europe without special permission; and the sea passage to the continent must be made within certain specified limits—usually the Texel and Brest. Some remarks will